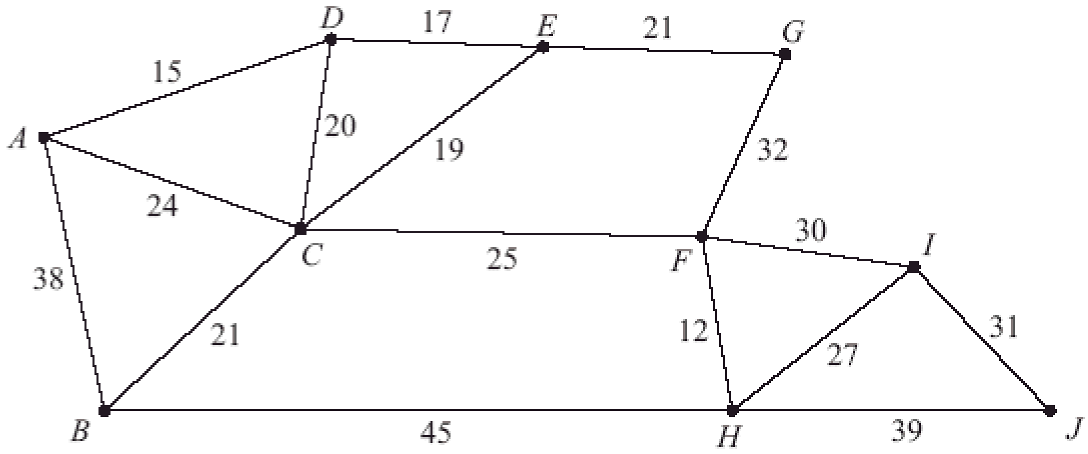
1.



The network in the diagram above shows the distances, in metres, between 10 wildlife observation points. The observation points are to be linked by footpaths, to form a network along the arcs indicated, using the least possible total length.

(a) Find a minimum spanning tree for the network in the diagram above, showing clearly the order in which you selected the arcs for your tree, using

(i) Kruskal's algorithm,

(3)

(ii) Prim's algorithm, starting from *A*.

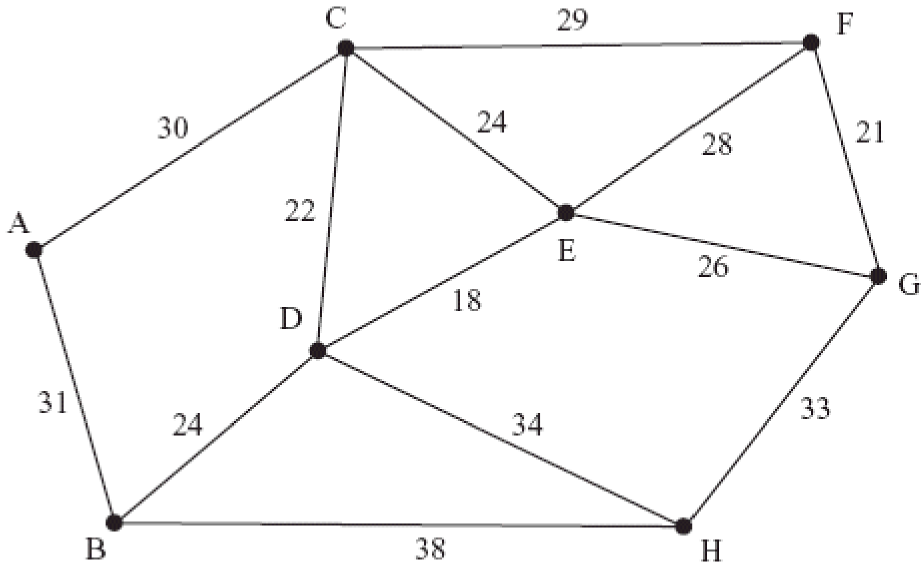
(3)

Given that footpaths are already in place along *AB* and *FI* and so should be included in the spanning tree,

(b) explain which algorithm you would choose to complete the tree, and how it should be adapted.
(You do **not** need to find the tree.)

(2)
(Total 8 marks)

2.



The diagram above represents the distances, in metres, between eight vertices, A, B, C, D, E, F, G and H, in a network.

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network.
You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(b) Complete the matrix below, to represent the network.

	A	B	C	D	E	F	G	H
A	–	31	30	–	–	–	–	–
B	31	–	–		–	–	–	38
C	30	–	–		24		–	–
D	–			–	18	–	–	
E	–	–	24	18	–	28		–
F	–	–		–	28	–	21	–
G	–	–	–	–		21	–	
H	–	38	–		–	–		–

(2)

(c) **Starting at A, use Prim's algorithm** to determine a minimum spanning tree. You must clearly state the order in which you considered the vertices and the order in which you included the arcs.

(3)

(d) State the weight of the minimum spanning tree.

(1)
(Total 9 marks)

3.

	A	**B**	**C**	**D**	**E**	**F**
A	–	24	–	–	23	22
B	24	–	18	19	17	20
C	–	18	–	11	14	–
D	–	19	11	–	13	–
E	23	17	14	13	–	21
F	22	20	–	–	21	–

The table shows the distances, in metres, between six vertices, **A**, **B**, **C**, **D**, **E** and **F**, in a network.

(a) Draw the weighted network using the vertices given in Diagram 1 below.

Diagram 1

(3)

(b) Use Kruskal’s algorithm to find a minimum spanning tree. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree.

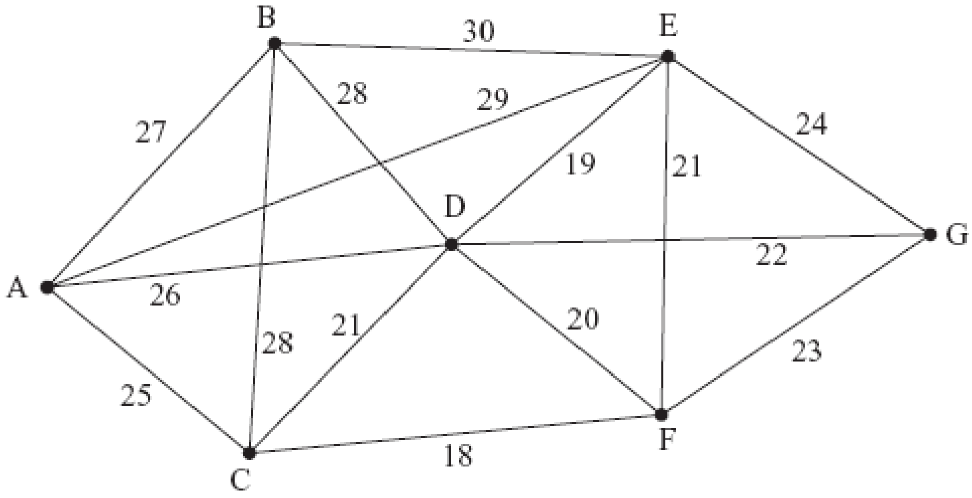
(3)

(c) Draw your tree on Diagram 2 below and find its total weight.

Diagram 2

(2)
(Total 8 marks)

4.



(a) State two differences between Kruskal's algorithm and Prim's algorithm for finding a minimum spanning tree.

(2)

(b) Listing the arcs in the order that you consider them, find a minimum spanning tree for the network in the diagram above, using

(i) Prim's algorithm,

(ii) Kruskal's algorithm.

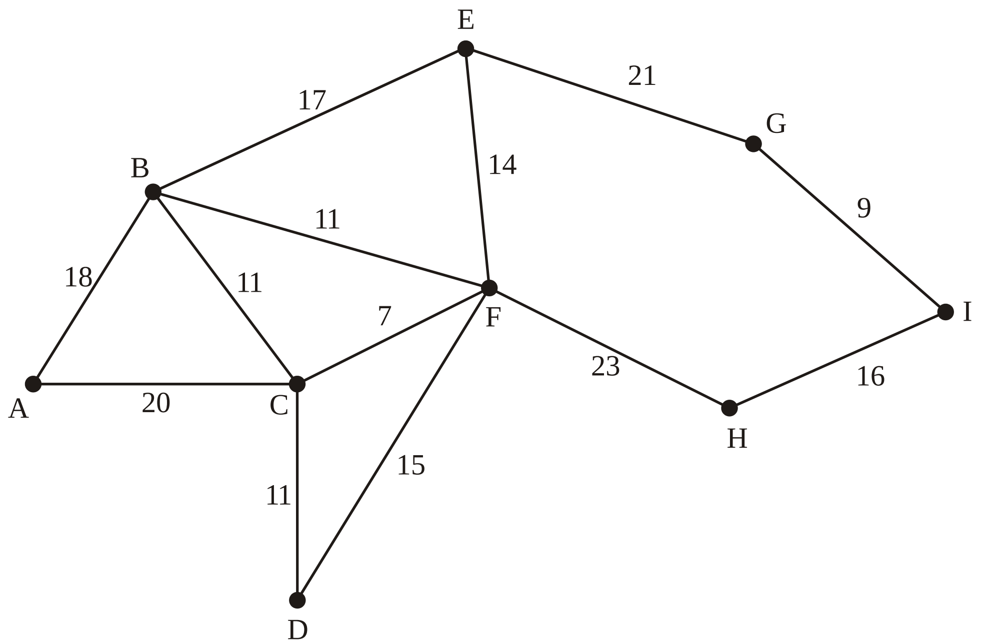
(6)
(Total 8 marks)

5. (a)

18 20 11 7 17 15 14 21 23 16 9

The list of numbers shown above is to be sorted into **ascending** order. Apply quick sort to obtain the sorted list. You must make your pivots clear.

(5)



The diagram above represents a network of paths in a park. The number on each arc represents the length of the path in metres.

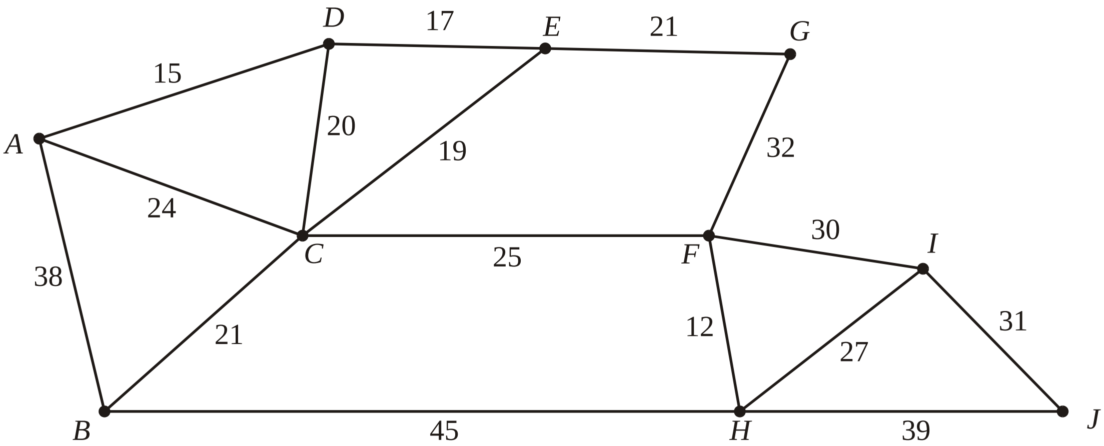
(b) Using your answer to part (a) and Kruskal's algorithm, find a minimum spanning tree for the network in the diagram above. You should list the arcs in the order in which you consider them and state whether you are adding it to your minimum spanning tree.

(4)

(c) Find the total weight of the minimum spanning tree.

(1)
(Total 10 marks)

6.



The network in the diagram above shows the distances, in metres, between 10 wildlife observation points. The observation points are to be linked by footpaths, to form a network along the arcs indicated, using the least possible total length.

(a) Find a minimum spanning tree for the network in the diagram above, showing clearly the order in which you selected the arcs for your tree, using

(i) Kruskal's algorithm,

(3)

(ii) Prim's algorithm, starting from *A*.

(3)

Given that footpaths are already in place along *AB* and *FI* and so should be included in the spanning tree,

(b) explain which algorithm you would choose to complete the tree, and how it should be adapted. (You do **not** need to find the tree.)

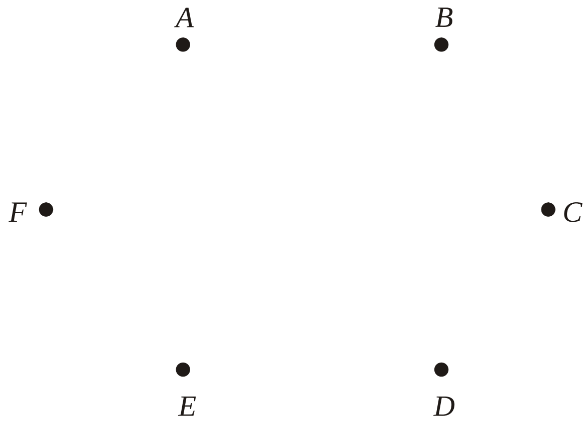
(2)
(Total 8 marks)

7.

	A	*B*	*C*	*D*	*E*	*F*
A	–	7	3	–	8	11
B	7	–	4	2	–	7
C	3	4	–	5	9	–
D	–	2	5	–	6	3
E	8	–	9	6	–	–
F	11	7	–	3	–	–

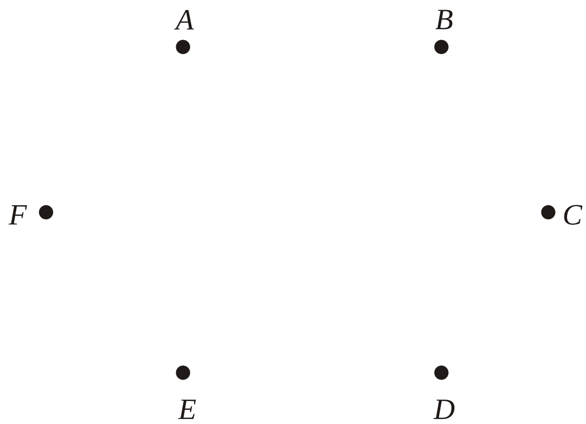
The matrix represents a network of roads between six villages *A*, *B*, *C*, *D*, *E* and *F*. The value in each cell represents the distance, in km, along these roads.

(a) Show this information on the diagram below.



(2)

(b) Use Kruskal's algorithm to determine the minimum spanning tree. State the order in which you include the arcs and the length of the minimum spanning tree. Draw the minimum spanning tree.



(5)

(c) Starting at D, use Prim's algorithm on the matrix given below to find the minimum spanning tree. State the order in which you include the arcs.

	A	B	C	D	E	F
A	–	7	3	–	8	11
B	7	–	4	2	–	7
C	3	4	–	5	9	–
D	–	2	5	–	6	3
E	8	–	9	6	–	–
F	11	7	–	3	–	–

(3)
(Total 10 marks)

8. (a) Define the terms

(i) tree,

(ii) spanning tree,

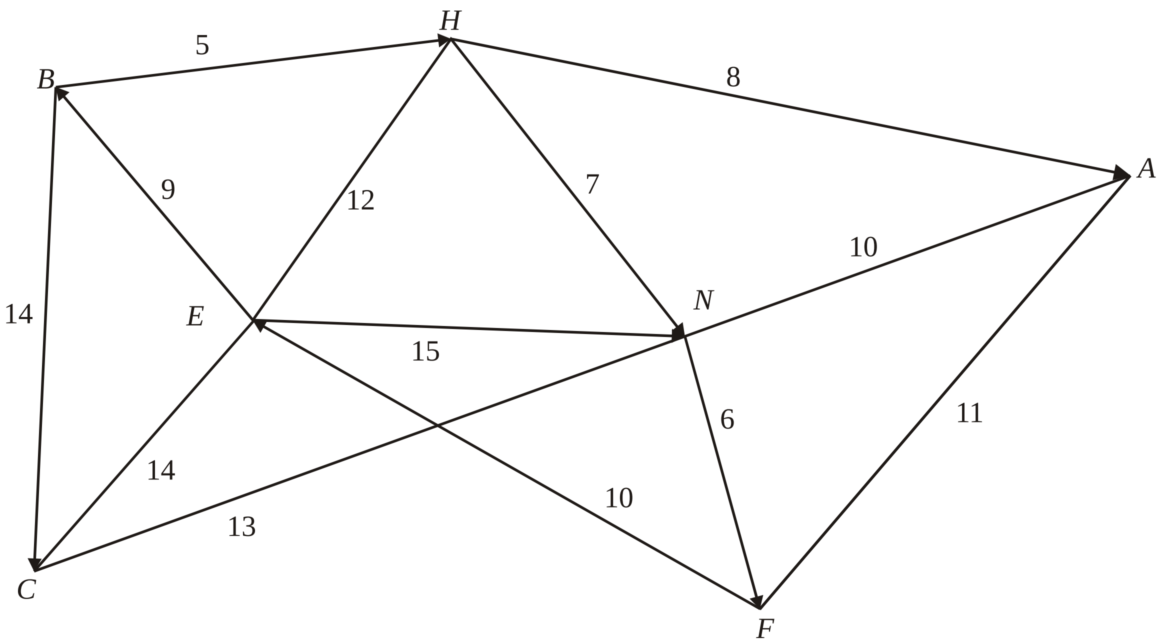
(iii) minimum spanning tree.

(3)

(b) State one difference between Kruskal's algorithm and Prim's algorithm, to find a minimum spanning tree.

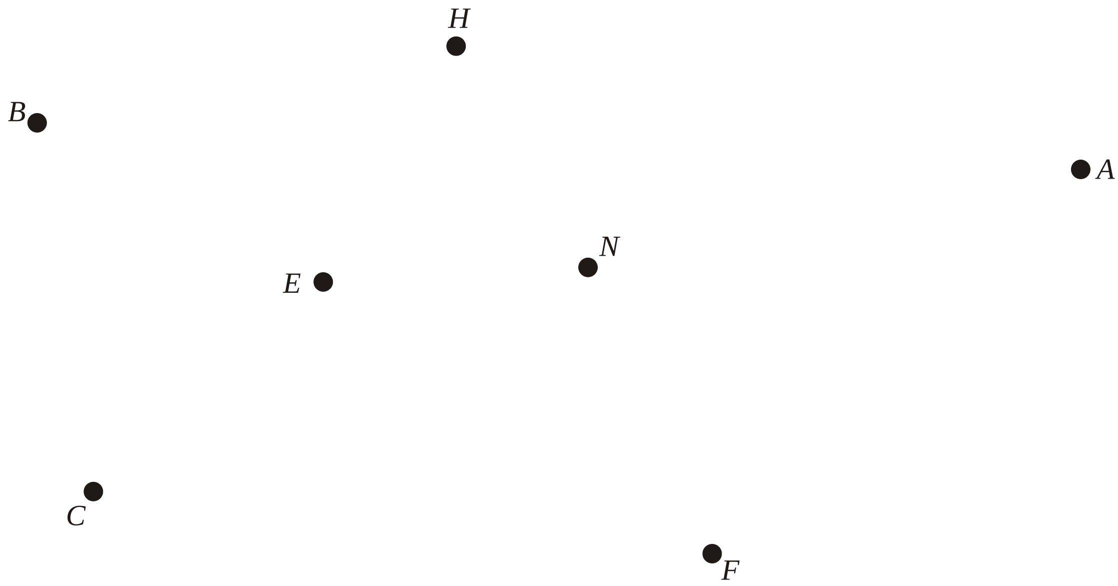
(1)

Figure 1



(c) Use Kruskal’s algorithm to find the minimum spanning tree for the network shown in Fig. 1. State the order in which you included the arcs. Draw the minimum spanning tree in Diagram 1 below and state its length.

Diagram 1



Order arcs included………………………………………………………………………………….

Length of minimum spanning tree………………………………………………………………...

(4)

Figure 2

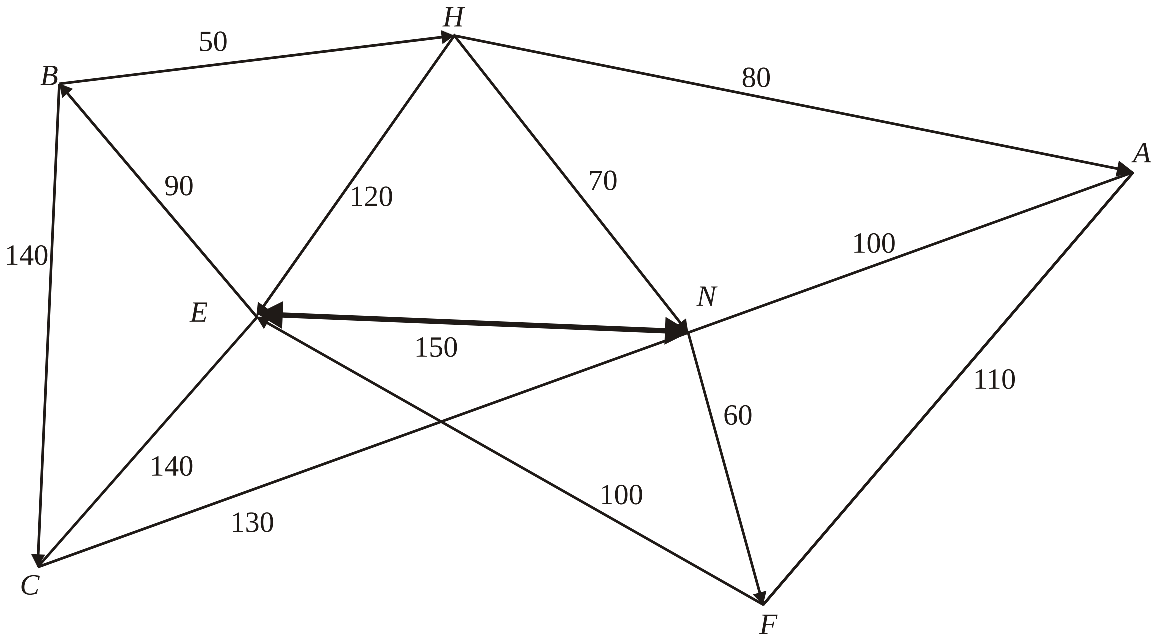
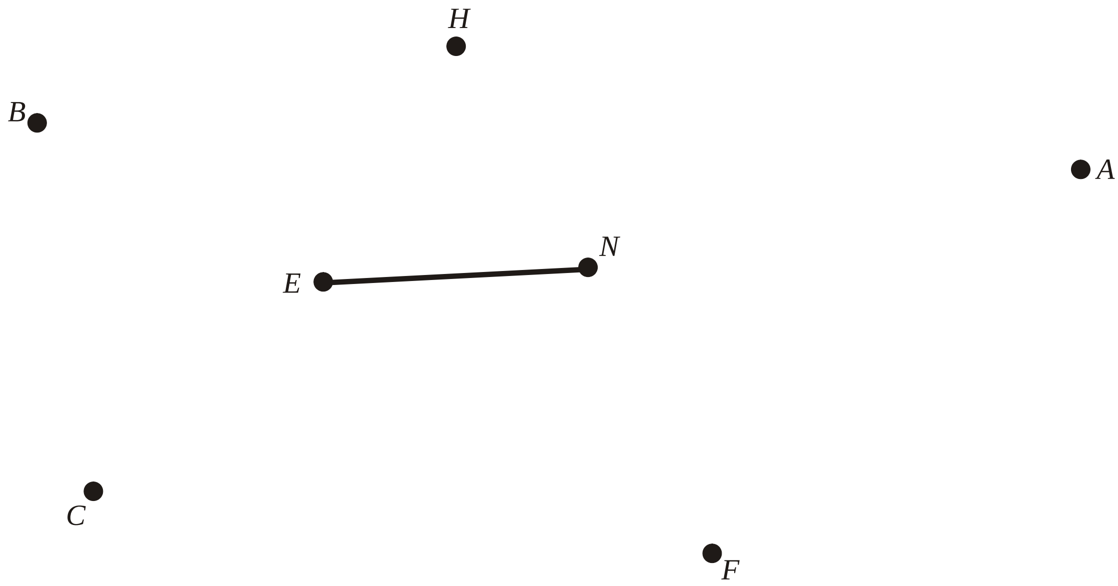

H
B
A
E
N
C
F
50
80
90
120
70
140
100
150
110
60
140
100
130

Figure 2 models a car park. Currently there are two pay-stations, one at E and one at N. These two are linked by a cable as shown. New pay-stations are to be installed at B, H, A, F and C. The number on each arc represents the distance between the pay-stations in metres. All of the pay-stations need to be connected by cables, either directly or indirectly. The current cable between E and N must be included in the final network. The minimum amount of new cable is to be used.

(d) Using your answer to part (*c*), or otherwise, determine the minimum amount of **new** cable needed. Use Diagram 2 to show where these cables should be installed. State the minimum amount of new cable needed.

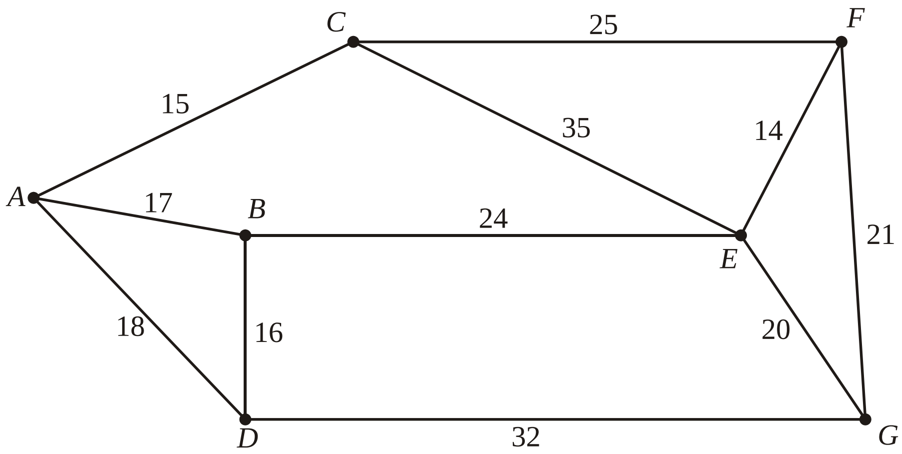
Diagram 2



(3)
(Total 11 marks)

9. (a) Describe the differences between Prim’s algorithm and Kruskal’s algorithm for finding a minimum connector of a network.

(3)



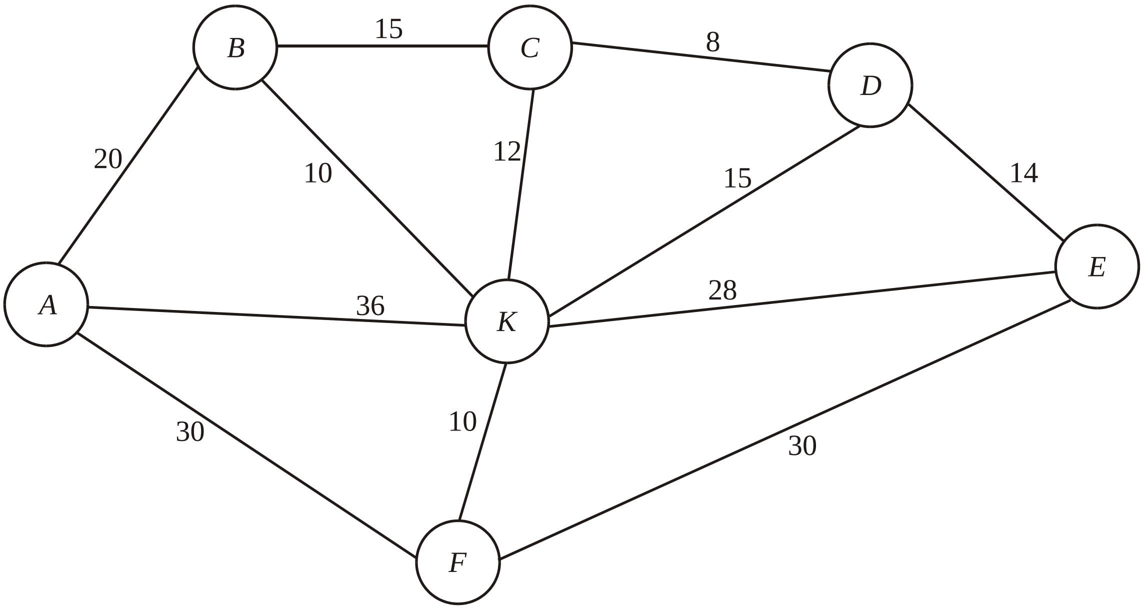
(b) Listing the **arcs** in the order that you select them, find a minimum connector for the network in the diagram above, using

(i) Prim’s algorithm,

(ii) Kruskal’s algorithm.

(4)
(Total 7 marks)

10. *This question should be answered on the page below.*



The diagram above shows a number of satellite towns *A*, *B*, *C*, *D*, *E* and *F* surrounding a city *K*. The number on each edge give the length of the road in km.

(a) Use Dijkstra's algorithm to find the shortest route from *A* to *E* in the network. Show your working in the boxes provided on the answer sheet.

(8)

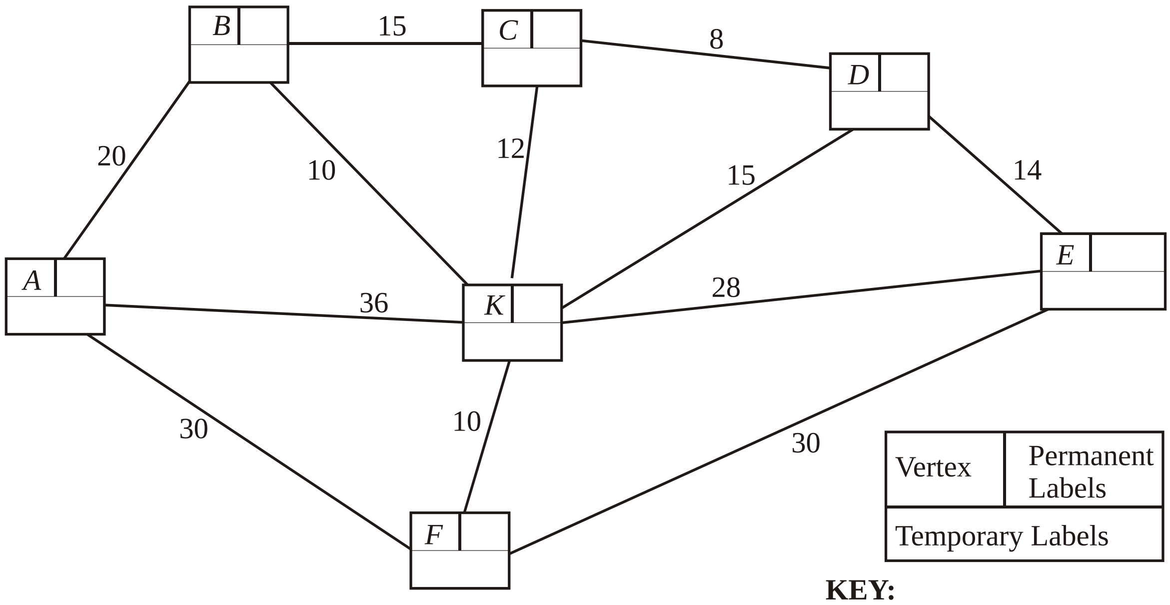
It is planned to link all the sites *A*, *B*, *C*, *D*, *E* and *F* and *K* by telephone lines laid alongside the roads.

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network and hence obtain the minimum total length of cable required. Draw your tree.

(6)
(Total 14 marks)

Sheet for use in answering this question

(a)



Length of Shortest Path:

....

Determination of shortest route:

....

....

....

....

....

(b) Minimum spanning tree

Total weight of minimum spanning tree: ....

1. (i) *FH*, *AD*, *DE*, *CE*, (not *DC*), $\begin{pmatrix} BC \\ EG \end{pmatrix}$, (not *AC*), *CF*, *HI*, (not *FI*), *IJ* M1A1A1 3

(ii) *AD*, *DE*, *EC*, $\begin{pmatrix} BC \\ EG \end{pmatrix}$, *CF*, *FH*, *HI*, *IJ* stop M1A1A1 3

(b) Start off the tree with *AB* and *FI*, then apply Kruskal M1 A1 2

[8]

2. (a) DE GF DC $\begin{Bmatrix} \text{not CE} \\ \text{BD} \end{Bmatrix}$ EG (not EF not CF) AC (not AB) GH M1 A1 A1 3

Note

1M1: Kruskal's algorithm – first 4 arcs selected chosen correctly.
1A1: All seven non-rejected arcs chosen correctly.
2A1: All rejections correct and in correct order and at correct time.

(b)

	A	B	C	D	E	F	G	H
A	-	31	30	-	-	-	-	-
B	31	-	-	**24**	-	-	-	38
C	30	-	-	**22**	24	**29**	-	-
D	-	**24**	**22**	-	18	-	-	**34**
E	-	-	24	18	-	28	**26**	-
F	-	-	**29**	-	28	-	21	-
G	-	-	-	-	**26**	21	-	33
H	-	38	-	**34**	-	-	**33**	-

B2, 1, 0 2

Note

1B1: condone two (double) errors
2B1: cao

(c) AC CD DE BD GE GF GH M1 A1 A1 3

Note

1M1: Prim's algorithm – first four arcs chosen correctly, in order,
or first five nodes chosen correctly, in order.{A,C,D,E,B….}
1A1: First six arcs chosen correctly **or** all 8 nodes chosen correctly, in order. {A,C,D,E,B,G,F,H}
2A1: All correct and arcs chosen in correct order.

(d) Weight: 174 B1 1

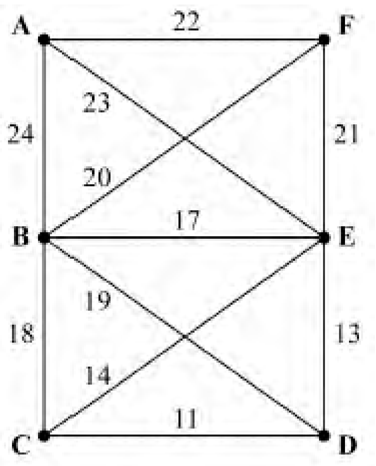
Note

1B1: cao

Starting at	Minimum arcs required for M1	Nodes	order
A	AC CD DE DB	ACDEB(GFH)	15234(768)
B	BD DE DC	BDEC(GFAH)	(7)1423(658)
C	CD DE DB	CDEB(GFAH)	(7)4123(658)
D	DE DC DB	DECB(GFAH)	(7)4312(658)
E	ED DC DB	EDCB(GFAH)	(7)4321(658)
F	FG GE ED DC DB	FGEDCB(AH)	(7)654312(8)
G	GF GE ED DC DB	GFEDCB(AH)	(7)654321(8)
H	HG GF GE	HGFE(DCBA)	(8765)4321

[9]

3. (a)



M1
A1
A1 3

Note

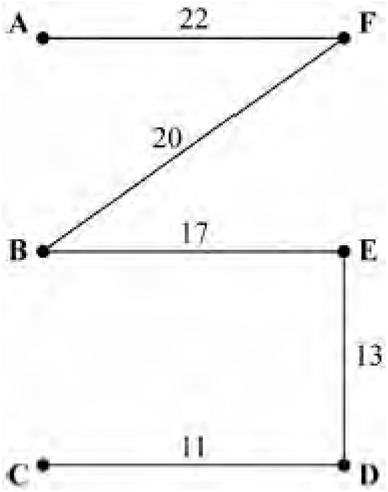
1M1: More than 10 arcs
1A1: all arcs correct
2A1: all values correct

(b) CD, DE, reject CE, BE, reject BC, reject BD, BF, reject EF, AF M1 A1
11 13 14 17 18 19 20 21 22 A1 3

Note

1M1: First three arcs correctly chosen
1A1: All used acrs selected correctly
2A1: All rejected arcs selected in correct order

(c)



	B1	
Weight of tree 83 (m)	B1	2

<u>Note</u>

1B1: CAO for arcs – numbers not needed. NO ft.
2B1: CAO 83, condone units

[8]

4. (a) e.g.

- Prims starts with any vertex, Kruskal starts with the shortest arc.
- It is not necessary to check for cycles when using Prim.
- Prims adds nodes to the growth tree, Kruskal adds arcs.
- The tree 'grows' in a connected fashion when using Prim.
- Prim can be used when data in a matrix form.

Other correct statements also get credit. B2,1,0 2

1B1: Generous one coprrect difference. If bod give B1

2B1: Generous two distinct, correct differences.

(b) (i) e.g. AC, CF, FD, DE, DG, AB. M1, A1, A1 3

(ii) CF, DE, DF, not CD, not EF, DG, not FG, not EG, AC, not AD, AB.
[18, 19, 20, not 21, not 21, 22, not 23, not 24, 25, not 26, 27]
M1, A1, A1 3

1M1: Prim's algorithm – first three arcs chosen correctly, in order, or first four nodes chosen correctly, in order.

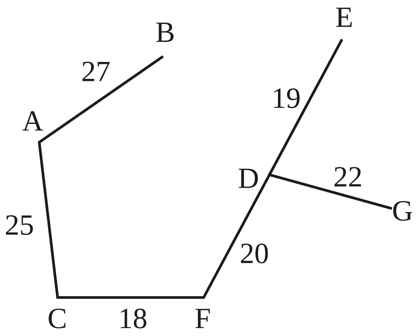
1A1: First five arcs chosen correctly; all 7 nodes chosen correctly, in order.

2A1: All correct and arcs chosen in correct order.

2M1: Kruskal's algorithm – first 4 arcs selected chosen correctly.

1A1: All six non-rejected arcs chosen correctly.

2A1: All regions correct and in correct order and ta correct time.



[8]

5. (a) e.g.

18	20	11	7	17	15	14	21	23	16	9	M1	
11	7	14	9	$\boxed{15}$	18	20	17	21	23	16	A1	
11	7	9	$\boxed{14}$		18	20	17	16	$\boxed{21}$	23	A1ft	
$\boxed{7}$	11	9			16	$\boxed{17}$	18	20		$\boxed{23}$	A1ft	
	$\boxed{9}$	11			$\boxed{16}$		18	$\boxed{20}$				
		$\boxed{11}$					$\boxed{18}$					
7	9	11	14	15	16	17	18	20	21	23	A1	5

(b) CF ✓ M1
GI ✓
{BC or BF(*) – accept one, reject one
{CD ✓ (*) A
EF ✓ (*)
DF ✗ (*)
HI ✓ (*) A1
BE ✗ (*)
AB ✓ (*)
AC ✗ (*)
EG ✓ (*)
Tree complete A1 4
(*) Needs letters

(c) 107 m — B1 1

[10]

6. (a) (i) FH,AD,DE,CE, (not DC), $\left\{\begin{matrix}BC\\EG\end{matrix}\right\}$, (not AC), CF, HI,

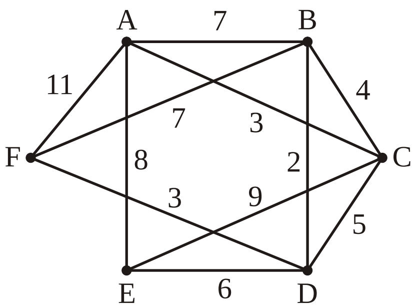
(not FI), IJ stop — M1 A1 A1 3

(ii) AD, DE, EC, $\left\{\begin{matrix}BC\\EG\end{matrix}\right\}$, CF, FH, HI, IJ stop — M1 A1 A1 3

(b) Start off the tree with AB and FI, then apply Kruskal — B2, 1, 0 2

[8]

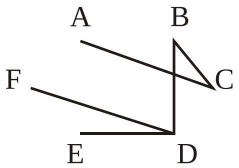
7. (a)



B1

B1 2

(b)



M1

BD, $\left(\frac{A\ C}{D\ F}\right)$, BC, Not CD, DE — A1, A1

B1

Length = 18 km — B1 5

(c) DB, DF, BC, CA, DE [5,2,4,1,6,3,] — M1 A1 A1 3

[10]

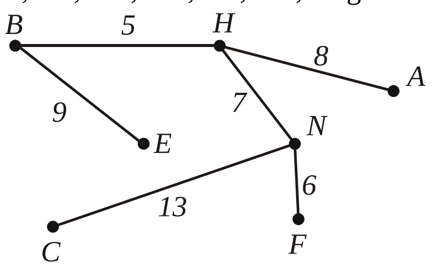
8. (a) (i) A connected graph with no cycles, loops or multiple edges — B1

(ii) A tree that includes all vertices — B1

(iii) A spanning tree of minimum total length — B1 3

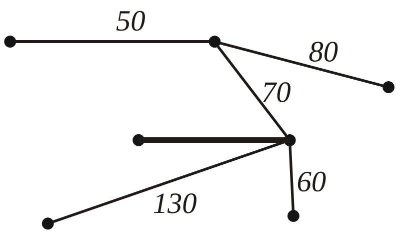
(b) E.g.
- In Kruskal the shortest <u>arc</u> is added (unless it completes a cycle), in Prim the nearest unattached <u>vertex</u> is added
- There is no need to check for cycles when using Prim, but there is when using Kruskal — B1 — 1
- In Prim the tree always "grows" in a connected fashion
- Kruskal starts with the shortest edge, Prim with any vertex

(c) BH, NF, HN, HA, BE, NC; length = 48 — M1 A1; B1



B1 — 4

(d)



B1

<u>New cable – 390m</u> — M1 A1 — 3

[11]

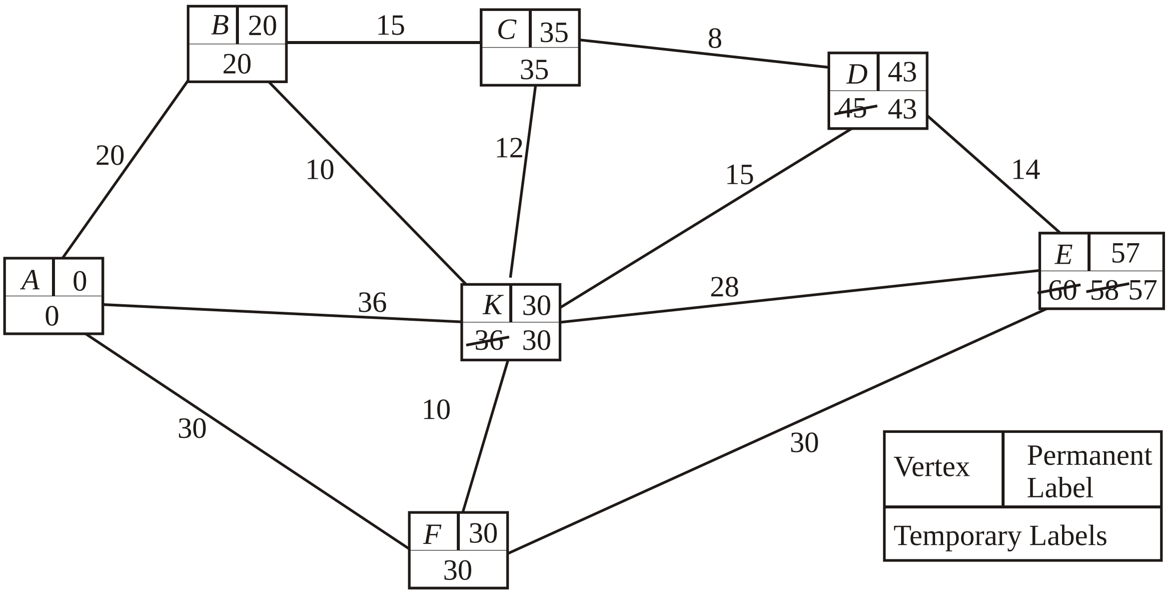
9. (a) e.g.:
- In Prim the tree always 'grows' in a connected fashion; — B3,2,1,0 — 3
- In Kruskal the shortest <u>arc</u> is added (unless it completes a cycle), in Prim the nearest unattached <u>vertex</u> is added;
- There is no need to check for <u>cycles</u> when using Prim;
- Prim can be easily used when network given in <u>matrix</u> form
- Prim <u>stats</u> with a <u>vertex</u>, Kruskal with an <u>edge</u>

(b) (i) Either *AC*, *AB*, *BD*, *BE*, *EF*, *EG* (if starts at *A* or *C*)
or *BD*, *BA*, *AC*, *BE*, *EF*, *EG* (if starts at *B* or *D*)
or *EF*, *EG*, *BE*, *BD*, *BA*, *AC* (if starts at *E* or *F*)
or *GE*, *EF*, *BE*, *BD*, *BA*, *AC* (if starts at *G*) — M1 A1

(ii) *EF*, *AC*, *BD*, *BA*, *EG*, *BE* — M1 A1 — 4

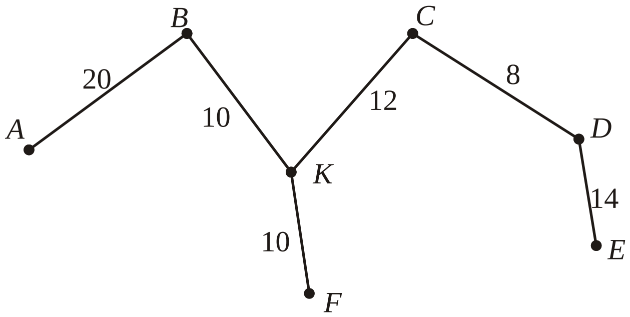
[7]

10. (a)



Length of Shortest Path: 57 km	B1
Determination of shortest route:	
Label E – Label D = 57 – 43 = 14 (DE)	
Label D – Label C = 43 – 35 = 8 (CD)	M1 A1
Label C – Label B = 35 – 20 = 15 (BC)	
Label B – Label A = 20 – 0 = 20 (AB)	
Hence shortest route is A B C D E	A1 8

(b) Minimum spanning tree



CD (8)
BK (10)
KF (10)
CK (12)
DE (14)

Not *KD* (15) Cycle
Not *BC* (15) Cycle
AB (20)

Total weight of minimum spanning tree: 74 km (tree) B1
(weight) B1 6

[14]

1. No Report available for this question.

2. Around 45% of the candidates gained full marks on this question. Candidates were directed to list the **arcs** in the order in which they included them in the tree, but many candidates did not do so.

In part (a) a number of candidates only stated the arcs they were including in their tree and did not state the arcs that they rejected, as they rejected them. Some candidates only referred to the length of the arc rather than by its end vertices, this makes it difficult for the examiners to determine which arc is being considered.

Part (b) was usually completed correctly.

In part (c) many candidates showed their working on the table but then did not list the arcs in the correct order, often adding BD too late.

Many candidates completed part (d) correctly.

3. Almost all the candidates gained full marks in part (a), with only a few ambiguous numbers on the diagonal arcs. There were also many fully correct solutions seen to part (b), although a few did not make the rejected arcs clear. Good presentation can be a great time saver here, many gave an ordered list of all arcs with just ticks and crosses by the rejections, others wrote long sentences about their decision to reject, due to cycles forming. Not all candidates stated the weight of the tree.

4. Part (a) caused problems for some candidates. Candidates were asked for differences between the two algorithms and many simply made statements such as Prim uses nodes without going on to say the Kruskal uses edges. A popular answer was to mention ordering the arcs into ascending order for Kruskal; if this used as a difference candidates must indicate why this is important when using the algorithm. There was the usual muddled use of basic technical terminology. Some candidates muddled the two algorithms. Part (b) as usually well-answered although some just drew their MST and did not list the arcs, in order, as instructed. Some wasted time in Prim by converting to the matrix form. Some did not make their rejections clear when applying Kruskal’s algorithm.

5. (a) Many excellent answers were seen here and the vast majority used the pivot selection seen on the mark scheme. There were however a number of flawed solution seen also. The most common errors were; random or inconsistent pivot choices; misordering of 21, 23, 16 on the first pass, misordering of 11 and 9 on the third pass and not selecting 20 as a pivot; only selecting one pivot per line. Some candidates used colour to indicate their pivots which should be discouraged. A very few bubble sorts were seen. In part (b) almost all the candidates were able to select the first two arcs, CF and GI but the three arcs of length 11 challenged many. There was often a lack of clarity concerning the order in which edges were rejected. Those who listed all the arcs, in order, and then ticked or crossed them gained the marks most efficiently. Some candidates wasted a great deal of time (and space) drawing numerous diagrams, each successive one having an extra arc.

6. There was a varied response to this question, with some very good and some quite poor responses seen. Part (a) was generally better attempted than part (b). In part (a) the candidates were asked to make the order in which they selected the arcs clear, many did not do this. Many candidates wasted time by drawing a succession of diagrams showing the addition of one arc at a time. Many candidates lost marks in Kruskal's algorithm by not showing the rejection of the arcs that created loops. In some attempts at Prim's algorithm, many did not list the arcs and others referred to rejecting arcs to prevent cycles from forming. Many wasted time in drawing a matrix for Prim. Part (b) was often poorly done with many candidates opting for Prim despite the two arcs not being connected. Of those who correctly selected Kruskal, only a few were able to give a coherent explanation.

7. Almost all of the candidates were able to draw the arcs correctly but it was not always clear which numbers went with which arcs, which lead to mistakes later. Most candidates were able to perform Kruskal's algorithm correctly although a number omitted to state that arc CD was rejected. Most went on to correctly draw the minimum spanning tree, but units were often omitted when stating its weight. Prim's algorithm was less well done. Some did not show that they had used the matrix, some started at A and many listed vertices instead of arcs.

8. The majority of candidates found part (a) tricky. There were some very poor definitions, poor use of technical terms and muddled thinking seen. Candidates often did not write in complete sentences, making it very difficult to understand the point they were trying to make. Part (b) was usually better done although candidates confused arcs, vertices, nodes and edges and made some worrying remarks about cycles. Part (c) was usually completed correctly. Part (d) caused problems for some candidates, with some incorrect diagrams and incorrect answers of 54m, 540m and 39m often seen.

9. Very few candidates attempted a comparison in part (a). Most just described the two methods. A significant number confused the two algorithms. There was some poor use of technical terms seen. Part (b) was very well done with most candidates gaining full marks, but a few did not list the arcs as requested. A surprising number of candidates seem to think that Prim's algorithm can only be applied to a matrix and wasted a lot of time in creating the matrix. Others wasted time (and often went onto at least two extra sides of paper) by drawing a 'cartoon sequence' showing the state of the tree after each arc addition, rather than simply listing the arcs in order.

10. No Report available for this question.